Modeling of the seismic response of models of asteroids based on the normal modes summation method

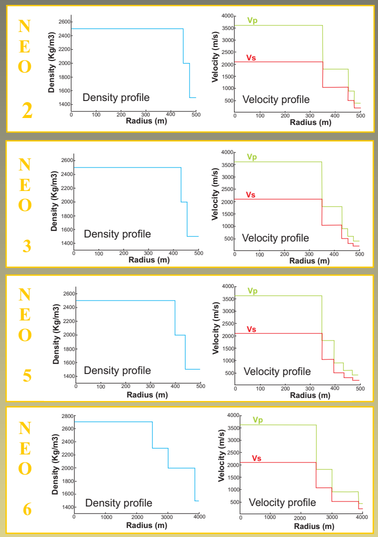
C. Blitz (1), P. Lognonné (1) and D. Komatitsch (2)

(1) Institut de Physique du Globe de Paris, Département de Géophysique Spatiale et Planétaire - (2) Université de Pau et des Pays de l'Adour, Laboratoire de modélisation et d'imagerie en Géosciences

OBJECTIVES

The development of possible mitigation techniques involves studies about the interior and the subsurface of asteroids, and as a consequence, experiments of seismological investigations are proposed. It is then necessary to define the characteristics a seismometer should have to image the interior of an asteroid. Two main features are then required: 1) the frequency range necessary to extract data about the internal structure of an asteroid, and 2) the instrument dynamics that defines the maximum amplitudes of the accelerations measured by the seismometer during an impact. To constrain these parameters, we need to compute seismograms on the surface of models of asteroids. The goal of this study is then to simulate the long period behavior of models of asteroids in response to a seismic source. The models used are spherical layered bodies, with diameters of 1 kilometer and 8 kilometers and the source is the one involved in the Don Quichotte ESA studies: an impact of a 400 kg projectile at a velocity of 10km/s (Ball et al., 2005). When the impact occurs, seismic waves propagate within the asteroid and are computed using the free oscillations summation technique. This method implies the computation of the normal modes of the models for angular order up to 500 and frequency bands ranging from 0 to 30 Hz (for models with 1 kilometer of diameter) and from 0 to 150 Hz (for models of 8 kilometers of diameter). As a second step, considering that a seismometer is located every 5° of epicentral distance (the source is at the pole of the model), we compute the 36 seismograms and 36 maximum values of accelerations on half of an asteroid (because the models are spherically symmetric). To accomplish this, the normal modes are summed, knowing that the test of a vertical impact force will induce spheroidal modes only (spheroidal modes corespond to Rayleigh surface waves with combined P and SV polarization). The simulation of a horizontal impact force will lead to both spheroidal and toroidal modes (toroidal modes are linked to the Love surface waves with SH polarization). The study of the maximum accelerations on the surface of the models allows us constraining the dynamics of a seismometer, and the computation of seismograms in given frequency bands are useful to evaluate the optimal bandwidth a seismometer should have to investigate a kilometer sized asteroid.

THE MODELS OF ASTEROIDS



The models of asteroids are spherical and layered, with diameters of 1 km (NEO2, NEO3, NEO5), and of 8 km (NEO6). The seismic velocities range from 200 to 3600 m/s with densities comprised between 1.5 and 2.7. Each model has a constant seismic quality factor of 1000, except for the model NEO2. Indeed, this model displays 2 versions: NEO2_1000 with a seismic quality factor of 1000 and NEO2_100000 with a seismic quality factor of 100000.

THE NORMAL MODES SUMMATION METHOD

To compute the normal modes of an asteroid, we have considered the characteristics of a spherical non-rotating elastic isotropic planet model, that leads to (Phinney and Burridge, 1973):

$$\sigma^2|u\rangle = A_0|u\rangle$$

σ: frequency
u: displacement of a particle
A_0: elastodynamic operator defined for a spherically symmetric model

The eigenfrequencies of this equation depend on 2 integer indices and can be noted $\omega_{n,l}$; the normal modes can be expressed by:

$$|u_{n,l,m}\rangle = U_{n,l}(r)Y_l^m(\Theta,\Phi) + V_{n,l}(r)\nabla Y_l^m(\Theta,\Phi) + W_{n,l}(r)e_r \wedge \nabla Y_l^m(\Theta,\Phi)$$

e_r: radial basis vector
r: position
n: radial order
l: angular order (positive integer)
m: azimuthal order such that $-l \leq m \leq l$
$Y_l^m(\Theta,\Phi)$: spherical harmonics functions

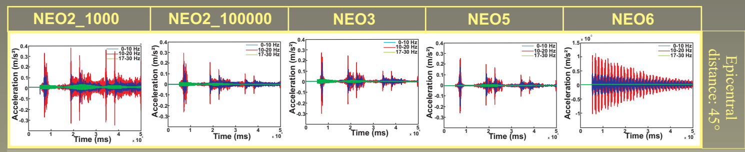
The expression of the displacement in spherical harmonics functions leads to 2 kinds of modes: spheroidal modes, that correspond to the functions U_K and V_K and toroidal modes corresponding to the function W_K. Assuming that the basis of normal modes is sufficient to describe displacements in a given seismic range, the seismogram of a planet in response to a seismic impulsive source $|S\rangle$ is expressed by the following equation:

$$|s(t)\rangle = \sum_{nlm} \langle R|u_{nlm}\rangle\langle v_{nlm}|s\rangle e^{i_n\sigma_l^m t}$$

${}_n\sigma_l^m$: common eigenfrequency of $|u_{nlm}\rangle$ and $\langle v_{nlm}|$ singlets
$\langle v_{nlm}|S\rangle$: coefficient of excitation at the source for a singlet *(n,l,m)*
$\langle R|$: instrumental response of the recorder
$\langle R|u_{nlm}\rangle$: coefficient of excitation at the station for a singlet *(n,l,m)*

RESULTS

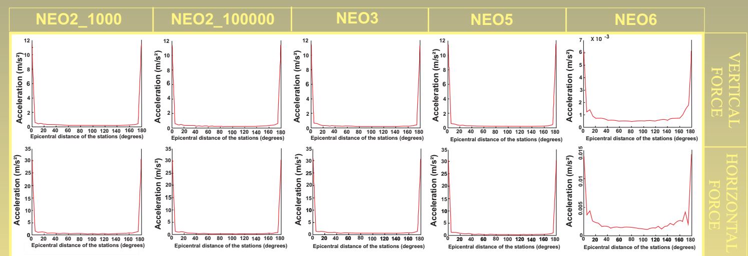
Frequency bands



Synthetic waveforms of horizontal component accelerations in several frequency bands (N componenet of the seimometer). The seismic source considered is the horizontal force of an impact F = 4.10^{11} g.cm.s^{-1}

To study the seismological properties of kilometers sized asteroids, the more appropriate frequency range would correspond to the bandwidth showing the maximum amplitudes of accelerations. this is strongly depending on attenuation and diffraction. Several seismograms have then been computed. The seismograms of the models NEO2 to NEO5 are clearly showing the different body waves R1, R2, ... successively arriving at the seismometer located at 45° of epicentral distance. This observation cannot be made on the largest model of asteroid (NEO6) since the effects of reverberation within a low velocity layer leads to a very different waveform. For each model, we observe maximum amplitudes located in the 0-10Hz and 10-20 Hz frequency bands. Furthermore,our programms show that frequencies up to 17Hz may force diffraction. The database of the modes displays the gravest modes at an average frequency of 1.2 Hz and 4.7 Hz respectively for asteroids of 1 kilometer and 8 kilometers of diameter. This study suggests anoptimal frequency band of 1-17 Hz for the seismological investigation of kilometer sized asteroids.

Maximum accelerations



Maximum accelerations computed for each model at each stations (separated by 5° of epicentral distance). A seismic source corresponding to an impact of a 400 kg projectile with a velocity of 10 km/s (F = 4.10^{11} g.cm.s^{-1}) is considered at an epicentral distance of 0°. Two cases of seismic sources have been studied: the effect of the vertical force of the impact (implying spheroidal modes only) expressed in the Z component of the seismometer, and the effect of the horizontal force of the impact (with both spheroidal and toroidal modes) computed in the N component of the seismometer.

These results show 2 main features: 1) the amplitudes of accelerations with respect to the epicentral distance (θ) exhibit a decrease and a refocusing of the surface waves that varies as 1/sqrt(sin(θ)) 2) the amplitudes of accelerations are more than twice stronger with a horizontal force as a seismic source than with a verticale force as source. This could be explained by the occurrence of both toroidal and spheroidal modes in the case of the horizontal force as seismic source. Finally, this study shows highest amplitudes of 30.2 m/s² and of $1.4.10^{-2}$ m/s² respectively for asteroids of 1 and 8 kilometers of diameter.

CONCLUSIONS

The representation of maximum accelerations on the surface of the asteroids models shows a refocusing of the seismic waves at the antipodes of the source. This location could be the place where the maximum acceleration could occur. The values of maximum accelerations at the surface of the models of asteroids impacted by a "Don Quichotte" type source are 30.2 m/s² and $1.4.10^{-2}$ m/s² respectively for asteroids of 1 and 8 kilometers of diameter. Our results have allowed an evaluation of the optimal frequency band for the study of kilometers sized asteroids. We find a range comprised between 1 and 17 Hz. However it should be mentioned that these values may significantly vary because seismic responses of asteroids are strongly dependant of their asphericity. The synthetic seismograms computed by the normal modes method for the models of asteroids presented here will be used for a comparison with synthetic seismograms of the same models that will be simulated by the spectral element method (Komatitsch et al., 2005). In a next step, the diffraction from both the surface and the interior of the asteroids models will be analyzed.

References: _Ball, A, Lognonné, P, Seiferlin, çt al. Lander and penetrator science for NEO mitigation studies, 2004. In "Mitigation of Hazardous Comets and Asteroids" edited by Belton, M. J. S., Morgan, T. H., Samarasinha, N and Yeomans, D.K. Cambridge University Press, 267-291, 2004; Phinney, R. A. And Burridge, R. Representation of the elastic-gravitational excitation of a spherical Earth model by geeralized spherical harmonics, Geophys. J. R. Astro. Soc., 34, 451-487, 1973_